Multiple-Structural Damage Detection using Measured Frequency Response Function

F. Homaei1, S. Shojaee2,* and G. Ghodrati Amiri1

1School of Civil Engineering, Iran University of Science & Technology, Tehran, Iran
2Department of Civil Engineering, Shahid Bahonar University of Kerman, Kerman, Iran

Received: 05 September 2014 Accepted: 22 May 2015

Abstract: Structural damage detection has been under attention since unknown damages make major hazard and case human losses. Researches in this field of study have been increased in the last few years. In this paper, a new method of damage detection in continues structures is presented. The proposed method is based on Frequency Response Function (FRF). Combination of measured FRF in a simple manner and using graphical diagrams make the user find the exact location of damaged element(s). According to finite element modeling of structures, a cantilever beam was modeled and the performance of the proposed method was checked. Decreasing the number of sensors and using rotational FRFs are the main advantages of the proposed method. Considering noise polluted data confirmed that this method can be used for practical purposes. In fact, using noise polluted data up to 5% doesn't have major effect on the results. The best excitation frequency of applied load is also studied in this research.

Keywords: Structural damage detection; Frequency response function (FRF); Finite element modeling; Damage detection; Degree of freedom

1. Introduction

Structural systems may be affected by some local damages during their lifetime. This can cause changing physical properties of structures like mass, stiffness and damping at damaged locations [1]. Using non-destructive test such as ultra-sound, visual inspection, liquid inspection and etc. can detect the defects; but, these methods are required long and expensive inspection time [2]. Because of the easier field performance of dynamic characterizing of structures than static one [3], using dynamic data for damage detection in more considered by the researchers.

Damage detection methods according to dynamic properties of structures have been studied by a number of researchers. A. Messina et al. [4] introduced the Multiple Damage Location Assurance Criterion (MDLAC) which is based on sensitivity and statistical method. Further, this criterion was developed and used by other researchers in damage detection process [5-8]. Using Optimization methods [6-10] (such as genetic algorithm and particle swarm optimization) and learning ones [12-15] (such as artificial neural network (ANN) and fuzzy neural networks) can be found in damage detection literatures. Although these method considered as effective ones in damage detection process, but time consuming and instability of solvers are problems that would be faced as the dimensions of the problems increase. Considering modal data based method in damage diagnosis problems [16] is another method that deals with modal response of structure and incomplete measured mode shapes and noise polluted data.

Using FRF data can be found in damage detection literatures [17-19]. Normalized imaginary part of FRF shapes was considered as a method of damage detection [17]. In that work, the imaginary FRF part was simplified by considering major effective parts and residual imaginary FRF was presented. Applying GSM and WT on residual imaginary FRF shape made magnification on damage indicative abnormality. FRF sensitivities by studying the influence of the number of natural frequencies, mode shapes, excitation location and the number of measured degree of freedom, was studied in laminated structures [18]. The problem of model incompleteness of damaged

* Corresponding author.
Associate Professor of Structural Engineering. E-mail address: saeed.shojaee@uk.ac.ir
structure was solved by using dynamic and static expansions of m-DOF of the damaged structure. Minimizing the difference between test and analytic FRF in order to detect the location and severity of structural damage by considering subset vectors of FRF was also studied. In order to measure FRFs, one needs to apply sensors to the structure. Using sensor data also have some limitations; for example, in order to decrease the expenses of the experiments, today, there is an effort to reduce the number of measuring sensors. Detection of damage presence in structures from changes in the natural frequencies is easier than damage localization. This is because of the same frequency changes of two different damage scenarios. In this paper, an improved method of damage detection, using measured FRF is proposed to detect the location of multiple damages in structures. Damage in structure is considered as reduction in elasticity modulus. Combination of FRF data of virgin and damaged structure in a simple manner is main purpose of this paper. Because of the difficulties and expenses of capturing all FRFs in practical purposes, only rotational FRF measurements under rotational excitations are considered as input data. The best excitation frequency is also studied. Decreasing the number of captured sensors, using rotational FRFs and considering noise polluted data are also mentioned in this research. Numerical finite element stimulations show satisfactory level of precision of the proposed method.

This paper consists of four sections. Theoretical formulation is presented in section 2. An overview of frequency response function (FRF) computation according to dynamic analysis and proposed damage detection method, are contents of this section in respect. In section 3, the advantages of the proposed method are studied by numerical stimulations. Computing the best excitation frequency, using noise polluted data and decreasing the numbers of sensors are also considered in this section. Conclusion around the whole method is the final section of this paper.

2. Theory

According to dynamic analysis theory, the equation of motion of an n degrees of freedom system is given as

\[
[M]_{n \times n} \{\ddot{x}(t)\}_{n \times 1} + [C]_{n \times n} \{\dot{x}(t)\}_{n \times 1} + [K]_{n \times n} \{x(t)\}_{n \times 1} = \{f(t)\}_{n \times 1}
\]

where \([M]_{n \times n}, [C]_{n \times n}\) and \([K]_{n \times n}\) are mass, damping and stiffness matrices, respectively. \(\{f(t)\}_{n \times 1}\) represents vector of applied force and \(\{\ddot{x}(t)\}_{n \times 1}, \{\dot{x}(t)\}_{n \times 1}, \{x(t)\}_{n \times 1}\) are vectors of acceleration, velocity and displacement of the structure. Considering a harmonic input, the applied force and displacement vectors can be expressed as

\[
f(\omega_j) = F(\omega_j) e^{j \omega_j t}
\]

\[
x(\omega_j) = X(\omega_j) e^{j \omega_j t}
\]

where \(\omega_j\) is the frequency of excitation force and \(j = \sqrt{-1}\). By substituting Eq. (2) and Eq. (3) in Eq. (1), the equation of motion can be rewritten as

\[
(-\omega_j^2 [M] + j \omega_j [C] + [K]) X(\omega_j) e^{j \omega_j t} = F(\omega_j) e^{j \omega_j t}
\]

From the above equation, the FRF matrices can be expressed as

\[
[H(\omega_j)] = (-\omega_j^2 [M] + j \omega_j [C] + [K])^{-1}
\]

Simplifying the above equation, the FRF matrices can be written as

\[
[H(\omega_j)] = [\text{FRF}(\omega_j)]= \begin{bmatrix}
H_{11}(\omega_j) & H_{12}(\omega_j) & \ldots & H_{1m}(\omega_j) \\
H_{21}(\omega_j) & H_{22}(\omega_j) & \ldots & H_{2m}(\omega_j) \\
\vdots & \vdots & \ddots & \vdots \\
H_{m1}(\omega_j) & H_{m2}(\omega_j) & \ldots & H_{mm}(\omega_j)
\end{bmatrix}
\]

\(H_q(\omega_j)\) in the above equation represents response of the structure at \(q\)th degree of freedom (DOF) when the excitation force is applied at \(j\)th DOF. Each element of \(H_q(\omega_j)\) is defined as

\[
H_q(\omega_j) = \sum_{\substack{i=1, \ldots, n \\\{j=1, \ldots, m\}}} \frac{\varphi_i \varphi_j}{(\omega_i^2 - \omega_j^2) + \left(2 \sum_{k \neq q} j \omega_k \omega_j \right)^2}
\]
where \( \varphi_k \) and \( \varphi_j \) are the \( i \)th and \( j \)th elements of \( k \)th mode shape \( \{ \varphi_k \} \) and \( \omega_k \) is the \( k \)th natural frequency and \( \zeta_k \) is the \( k \)th damping ratio. As in this paper, the main objective is to identify the location of structural damage, which is related to the stiffness matrices, the damping ratio is neglected even though damping reduces the natural frequency. So damping matrices of virgin and damaged structures are considered the same. The same strategy is used for mass matrices.

Measuring all FRFs are time and cost consuming. So as it was mentioned before, in this research; in order to decreasing measured data, only rotational FRFs are considered as input data. The excitation forces are also applied to rotational DOFs. So when excitation force is applied at \( (2 \times j) \)th DOF, only rotational responses of structure are captured by sensors. If transition DOFs are considered as odd rows and columns of FRF matrices and rotational DOFs as even ones, Eq. (6) can be simplified as

\[
\begin{bmatrix}
H_{22}(\omega_j) & H_{24}(\omega_j) & \cdots & H_{2(2m-1)}(\omega_j) \\
H_{42}(\omega_j) & H_{44}(\omega_j) & \cdots & H_{4(2m-1)}(\omega_j) \\
\vdots & \vdots & \ddots & \vdots \\
H_{(2n-2)2}(\omega_j) & H_{(2n-2)4}(\omega_j) & \cdots & H_{(2n-2)(2m-1)}(\omega_j)
\end{bmatrix}
\]

(8)

Summing up the rotational FRFs rows yields

\[
\{SH_k(\omega_j)\} = \left\{ \sum_{j=1}^{\infty} H_{2j}(\omega_j), \sum_{j=1}^{\infty} H_{4j}(\omega_j), \cdots, \sum_{j=1}^{\infty} H_{(2n-2)j}(\omega_j) \right\}^T
\]

(9)

After computing the \( \{SH_k(\omega_j)\} \) vector for virgin and damaged structures, the result can be substituting in Eq. (10)

\[
\{DI\} = \exp\left( \left[ \{SH_k(\omega_j)\}^{\text{Damaged}} \right]^{\frac{1}{2}} - \left[ \{SH_k(\omega_j)\}^{\text{Virgin}} \right]^{\frac{1}{2}} \right) \quad (n \in N)
\]

(10)

where \( \{DI\} \) represents Damage Index vector. By drawing \( \{DI\} \) vector in a bar diagram and specifying sudden rises, damaged locations will be represented in structure. Here, the value of \( n \) in Eq. (10) plays a role in better representing the results. The following section will represent numerical stimulations that make clearance about the performance of the proposed method.

### 3. Numerical Simulation

In this section, the proposed damage detection method applied to a beam structure. A cantilever beam (Fig. 1.) with 50 elements is subjected to damages and diagnosis process would be applied according to the proposed method. The FRFs can be obtained from experimental data, but in this work, FRFs are obtained according to numerical analysis of both virgin and damaged beam by using finite element methods. The best load excitation frequency is considered as one of the natural frequency of the virgin structure. In order to make a close relation between numerical analysis and measured data, noise polluted data are considered in section 3.2. Also, decreasing the number of sensors is considered in that section. Beam physical and mechanical properties are shown in Table 1. Damage scenario is represented in Table 2.

![Fig. 1. A cantilever beam with 50 elements](image)

Table 1. Beam section properties

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Description</th>
<th>Modulus of Elasticity ( E ) (Kg/cm(^2))</th>
<th>Mass unit of Volume ( \rho_e ) (Kg/cm(^3))</th>
<th>Section Height ( h ) (cm)</th>
<th>Section Width ( b ) (cm)</th>
<th>Element Length ( l_e ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cantilever beam</td>
<td>2100000</td>
<td>7.85E-3</td>
<td>2.50</td>
<td>2.50</td>
<td>1.00</td>
</tr>
</tbody>
</table>


Table 2. Damage scenarios of the beam structures

<table>
<thead>
<tr>
<th>Damage scenario of the 50 elements cantilever beam</th>
<th>Damage ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damaged elements No.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15%</td>
</tr>
<tr>
<td>11</td>
<td>20%</td>
</tr>
<tr>
<td>17</td>
<td>10%</td>
</tr>
<tr>
<td>25</td>
<td>5%</td>
</tr>
<tr>
<td>33</td>
<td>25%</td>
</tr>
<tr>
<td>47</td>
<td>10%</td>
</tr>
</tbody>
</table>

3.1. Example 1: Analytical Models without Noisy Data

3.1.1. The Best Excitation Frequency

In order to measure FRFs, excitation forces should be applied to DOFs. These forces excite the structure under a specified frequency. In this work, the authors proposed the following method:

1. Measure all rotational natural frequencies of the virgin beam (according to analytical method).
2. Compute the Natural Frequency Criterion (NFC) according to the below equation:
   \[ NFC_{i} = \sqrt{\frac{\sum_{i=1}^{n} (f_{oi}^{2} - (f_{i}^{2})^{2})}{n \times \text{Number of Rotational DOFs}}} \] (11)
3. Draw the computed values of NFC in a bar diagram.
4. Specify the first point which is proportional relative minimum.
5. Use proportional frequency of specified point as the excitation frequency.

So, for the beam examples, the above procedure was done and the results are shown in Fig. 2.

![Fig. 2. Rotational natural frequency of beam models](image)

As it can be seen, the specified frequency (25th rotational natural frequency) is considered as the best excitation frequency that can be used in this method.

3.1.1. Damage Detection

After computing rotational FRFs (under rotational excitements) for both damaged and virgin structures, the \(\{SH_{a}(\omega)\}\) vectors computed. Substituting these vectors in Eq. 10, the \(\{DI\}\) vector can magnify damaged sites by visualizing it in a bar diagram. In this section, all the rotational DOFs responses are used. As it can be seen from Fig. 4(a), sudden rises of this diagram represent the nodes number of damaged elements. It should be noted that for these analysis, \(n=2\) considered in Eq. (10). Another way that can be helpful in some situations (for example doubting about sudden rises of \(\{DI\}\) diagram) is to draw the Damage Index vector differences of neighbor nodes. In this procedure, the \(\{DI\}\) vectors of two side nodes are subtracted and the results are shown on a bar diagram (Fig. 4(b)). In this diagram, sudden dropping from positive values to negative ones represents damage presence.
It should be mentioned that for a cantilever beam, as the DOFs of free end of the beam have no restriction, a liar value can be seen on the diagram. Here, node number 51 represents damage presence in element number 50. Looking accurately through the whole diagram shows that for an element to be specified as a damaged one, both initial and end DOFs should mention damage presence according to \( DI \) diagram. So for concluding damage presence in element No.50, both 50 and 51 DOFs should specify maximum values. But here, only DOF No.51 represents relative maximum and that means, element No.50 is not a damaged one.

![Damage detection process by visualizing \( DI \) vector of cantilever beam](image1)

Fig. 3. Damage detection process by visualizing \( DI \) vector of cantilever beam (a) Absolute \( DI \) vector (b) Damping index vector differences

### 3.2. Example 1: Analytical Models with Noise Polluted Data

In all experimental data, there exist errors. To stimulate these experimental inaccuracies, random error is added to the measurements. In this section, the performance of the proposed method is investigated by considering noise polluted data. This study is continued by adding 5% uniform random error to the FRFs of damaged structure and considering natural frequencies to be noise-free. Reducing the captured sensors is also studied in this section.

For the cantilever beam, 10 measured FRFs of 10 sensors (about 20% of nodes) are used to compute \( DI \) vector. Also for better visualization, \( n=5 \) considered in Eq. (10). The excitation frequency is the same as the one computed in section 3.1.1. The results are shown in Fig. 5. As it can be seen, applying noise to measured data does not have much influence on the proposed method. It should be mentioned that by increasing the number of sensors, better results can be obtained. The results of using all DOFs measured data are also shown in Fig. 6.

![Damage detection process by visualizing \( DI \) vector of cantilever beam using noise polluted data and measured FRFs of \( \{5,10,15,20,25,30,35,40,45,50\} \) DOFs](image2)

Fig. 4. Damage detection process by visualizing \( DI \) vector of cantilever beam using noise polluted data and measured FRFs of \( \{5,10,15,20,25,30,35,40,45,50\} \) DOFs (a) Absolute \( DI \) vector (b) Damage Index vector differences

![Damage detection process by visualizing \( DI \) vector of cantilever beam using noise polluted data and all measured FRFs](image3)

Fig. 5. Damage detection process by visualizing \( DI \) vector of cantilever beam using noise polluted data and all measured FRFs (a) Absolute \( DI \) vector (b) Damage Index vector differences
4. Conclusions

A new method of multiple damage detection in structures using measured FRFs has been proposed. In this method, only rotational FRFs under rotational excitements are considered. The measured values are combined in a simple manner and make Damage Index vector. Drawing this vector in a bar diagram make a visual background to specify damaged element(s). The performance of the proposed method was also checked by applying 5% uniform random error to the FRFs of damaged structure. Decreasing the number of measuring sensors, make this method as an operable one for practical purposes. Therefore, the proposed method can be assumed as a new, simple and useable method of multiple damage detection according to FRFs.

References


